QUIZ 12 SOLUTIONS: LESSONS 17-18
OCTOBER 13, 2017

Write legibly, clearly indicate the question you are answering, and put a box or circle around your final answer. If you do not clearly indicate the question numbers, I will take off points. Write as much work as you need to demonstrate to me that you understand the concepts involved. If you have any questions, raise your hand and I will come over to you.

1. [6 pts] Suppose an ant is walking towards her colony which is 3.21 feet away. Suppose in the first minute she walks $\frac{3}{4}$ths of a foot, in the next minute she walks $\frac{9}{16}$ths of a foot, in the next minute she walks $\frac{27}{64}$ths of a foot, and she continues this pattern indefinitely. Will she ever make it back to her colony? Give a reason for your answer.

   Solution: This describes a geometric series. In the first minute, the ant travels $\frac{3}{4}$ths of a foot. In the next, the ant travels $\frac{9}{16} = \frac{3^2}{4^2}$ths of a foot. In the next minute, the ant travels $\frac{27}{64} = \frac{3^3}{4^3}$ths of a foot. So the ant travels $\frac{3^n}{4^n} = \left(\frac{3}{4}\right)^n$ths of a foot in the n^{th} minute. Thus, our series is given by

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n.$$

   Now, this series converges because $\left|\frac{3}{4}\right| < 1$. So we can apply the geometric series formula, although this isn't quite in the correct form. We write

$$\sum_{n=1}^{\infty} \left(\frac{3}{4}\right)^n = \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)^{n+1} = \sum_{n=0}^{\infty} \left(\frac{3}{4}\right)\left(\frac{3}{4}\right)^n = \frac{\frac{3}{4}}{1-\frac{3}{4}} = \frac{\frac{3}{4}}{\frac{1}{4}} = 3.$$

   What does this mean? It means that given all the time in the world, this ant will only travel at most 3 feet. Therefore, she will **not** make it back to her colony.

2. Let $f(x, y) = \dfrac{\ln(3x+1)}{\sqrt{3x-y}}$.

   (**a**) [3 pts] Find the domain of $f(x, y)$. Put your answer in set builder notation.

Solution: We need to check 3 things for the domain of a function:

(1) No dividing by zero

We can't have $\sqrt{3x-y}=0 \Rightarrow 3x-y\neq 0$.

(2) Even roots have non-negative input

We have to have $3x-y\geq 0$.

(3) ln has positive input

$3x+1>0$

Putting all of this together, our set is given by

$$\boxed{\{(x,y) : 3x-y\neq 0, 3x-y\geq 0, 3x+1>0\}}$$

An equivalent way to write this is

$$\boxed{\{(x,y) : 3x-y>0, 3x+1>0\}}$$

(**b**) [1 pt] Find $f\left(\frac{e^2-1}{3}, e^2-5\right)$

Write

$$\begin{aligned} f\left(\frac{e^2-1}{3}, e^2-5\right) &= \frac{\ln\left(3\left(\frac{e^2-1}{3}\right)+1\right)}{\sqrt{3\left(\frac{e^2-1}{3}\right)-(e^2-5)}} \\ &= \frac{\ln(e^2-1+1)}{\sqrt{e^2-1-e^2+5}} \\ &= \frac{\ln(e^2)}{\sqrt{4}} \\ &= \frac{2}{2} = \boxed{1} \end{aligned}$$